Construction of a Common Errors Database in Mathematics for Intelligent Tutoring System Development

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***Abstract*—Mathematics is one of the most important subjects in computing and engineering educational curricula worldwide. However, many students face different kinds of difficulties in this discipline. In this context, Intelligent Tutoring Systems (ITS) can provide an environment that supports students' learning, for instance, by identifying learning problems, which can be done based on error patterns. Nevertheless, creating error bases is costly and difficult to access. Thus, this work presents an exploratory study that investigated the tests of 63 mathematics students from public schools and mapped a set of common errors that will serve as a common basis for the construction of ITS to be applied to mathematics**

Keywords—Common Errors, Intelligent Tutoring System (ITS), Math

I. Introduction

Mathematics is one of the most important subjects for personal and intellectual development. However, various difficulties exist in the daily lives of math students and teachers worldwide. In this context, techniques, methodologies, and tools are proposed to minimize these difficulties. Among them, Intelligent Tutoring Systems (ITS) stands out as a digital tool that provides learners with an individual and tailored experience, offering a comprehensive environment for students to explore and learn various content.

However, identifying errors to provide feedback and personalize responses can be challenging in ITS development. Therefore, building a database to enable ITS to identify student errors and to assist students' learning is a priority in modeling mathematics-oriented ITS.

This exploratory study aims to gather relevant data to map common errors in mathematics subjects. Through this data, models of ITS capable of autonomously identifying, predicting, and creating errors can be developed. These models, in turn, can be applied to construct solutions for mathematics education.

The data obtained in this study aims not only to identify common errors but also to create methods for dynamically replicating them. As this study is part of a larger research project focused on constructing ITS for mathematics, the data collected can serve as a source for other researchers to acquire information about common errors in mathematics, particularly those applied to ITS.

In this context, to achieve the study's objective, the following hypotheses (H) were formulated:

- ***H1****: It is possible to identify common errors and organize them in a structured manner, facilitating the creation of solutions based on the data.*
- ***H2:*** *Common errors will be present in tests at similar levels, as well as errors related to transient difficulties, acquired at more basic stages of education up to the current level the student is attending.*

Thus, the following sections are organized to present the findings of this study, comprising a theoretical foundation section, a presentation of the applied methodology, another presenting the findings, and finally, the discussion and conclusion of this study.

II. Literature Review

For this study, two concepts are very important, the use of distractors in tests and ITS. Distractors are stimuli or elements that interfere with students' attention and concentration, impairing their ability to assimilate information effectively [1]. Thus, when introduced in a controlled environment, the presence of distractors can have a significant impact on students' learning process [1].

In the context of assessments, a distractor can be understood as an element that intentionally leads the student to an incorrect response and is associated with common errors that students frequently encounter throughout their academic lives [2]. These distractors manifest themselves through multiple-choice alternatives, consisting of one correct option and the other options referred to as distractors [2].

In Brazil, the use of distractors gained visibility through the National High School Exam (ENEM), which, starting in 2009, after a reformulation of the exam structure, began to adopt the use of distractors in the response options. ENEM is a test that assesses the knowledge and skills that students have acquired throughout high school for admission to higher education in Brazil [3].

According to Larkin et. al. [4], when a novice student is exposed to a question prompt, they search for elements that can answer it, associating these elements with the proposed options. In this context, distractors can be developed to represent incorrect associations and misconceptions based on the question prompt, leading students who do not fully understand the content to choose the wrong answer.

In addition to distractors, the other fundamental concept in this study is ITS, Intelligent Tutoring Systems, computer systems that aim to provide personalized and adaptive support for learning in a specific domain. These systems combine techniques of Artificial Intelligence (AI), such as machine learning algorithms and knowledge models, with pedagogy to create an effective and interactive learning environment [5].

A typical ITS consists of several interconnected components. These components include [6]:

- Student Model: Responsible for representing the student's knowledge, progress, and difficulties. It can be built based on cognitive models or through data mining techniques;
- Domain Model: Represents knowledge about the specific learning domain, including concepts, rules, and problem-solving strategies;
- Teaching Model: Provides information on how to teach the domain concepts to students, including instructional sequences and pedagogical strategies;
- Student Interface: The interface through which the student interacts with the system, which can be a graphical, voice, or even virtual interface;
- Feedback Mechanism: Provides feedback to the student on their performance, identifying errors and offering correction suggestions.

ITS offers benefits in the teaching and learning process, such as [6]:

- Personalized Learning: ITS adapts content and activities according to the individual needs of each student, allowing for personalized and efficient learning;
- Immediate Feedback: Students receive immediate feedback on their performance, enabling them to correct errors and improve their learning more quickly and effectively;
- Continuous Monitoring: ITS monitors the student's progress over time, identifying areas where the student faces difficulties and providing specific interventions to overcome these challenges;
- Student Engagement: ITS uses interactive techniques and multimedia resources to engage and motivate students, making the learning process more attractive and engaging.

III. Methodology

This section presents the applied methodology of this study, including data acquisition, manipulation, and structuring.

A. Data Acquisition Process

Answers from three different tests given to 63 K12 Brazilian students were obtained. The researchers involved in this study had access to the raw test data, including math question statements and step-by-step students' answers with all calculations.

The test consisted of 6 objective questions from well-known public exams in Brazil, given to 11 students from two first-year high school classes. Questions 1 to 5 were regular (medium difficult level) exam questions, while Question 6 was a bonus challenge (high difficult level) that could add additional points to the overall score. All questions consisted of five alternatives, with only one correct option and four distractors. The test covers topics such as the Theorem of Thales, Pythagorean Theorem, and the rule of three.

The second test comprised 4 questions created by Math teachers for this study and administered in a class with 20 students who took the tests in pairs (10 collected tests in total). Questions 1 to 3 were closed-ended with five alternatives (one correct answer and five distractors). The distractors try to lead the student to make mistakes due to common learning issues. Finally, question 4 consisted of 3 open-ended sub-questions, each one requesting a specific math operation. This test covered concepts related to spatial geometry, given to students of a K12 integrated technical course.

The third test was administered similarly to the second, with pairs of students taking the tests, totaling 32 students in 16 groups. This exam assessed concepts of modular and exponential functions, comprising 10 questions. Questions 1 and 5 to 10 used distractors in their alternatives, with 1 correct answer and 5 distractors in each. Questions 2 to 4 were open-ended. Questions 1 to 4 explored the concepts of modular functions, while the rest covered exponential function concepts.

To preserve the anonymity of the students, all student identifications were removed before sending the tests to the researchers of this study. It is believed that scratch paper may have been provided to students during the tests. Hence, it was explicitly mentioned that all calculations should be written in the test paper, resulting in point deductions in cases where students provide only the final response, even if they got the answer right.

B. *Pre-Processing and Data Processing Process*

After obtaining the tests, they were analyzed to structure the data for the processing phase. Firstly, labels were created to identify each test. The labels were represented by uppercase letters in ascending order, starting with the letter A for the first set of students and numbers, starting with 1, for the second set of tests. Thus, the tests from the first set were classified into Group A, with 4 tests labeled A, B, C, and D, and Group B, containing 7 tests labeled K, L, M, N, O, P, and R. This order is associated with the arrangement of the tests in two separate files called "Turma A" and "Turma B", respectively.

In turn, the tests from the second and third sets were labeled with consecutive numbers as arranged in the provided document, a set of sheets organized in PDF format. In total, 10 tests were applied in set two, and 16 tests in set three, as the tests were done in pairs. Thus, each test received a numerical label according to the order in which it appears. In this way, test 1 includes the test of students 1 and 2, test 2 of students 3 and 4, and so on.

The next step was the identification, labeling, and description of errors. In this stage, a senior undergraduate student majoring in Mathematics analyzed each question and labeled them as correct, partially correct, or incorrect, followed by a description of the errors found in the students' answers.

After the analysis and structuring of the data, a second analysis was conducted by the main researcher of this study to identify patterns based on the described data, thus concluding the data analysis phase of the tests and preparing them for the post-processing stages to be presented in the following chapters.

C. *Findings*

This section presents the results of the findings after the structuring of the data obtained from the tests. This chapter is divided to first present the findings of the first set of tests from the classes of the state school in RN, and then two other sections will present the data obtained from the analysis of tests from the Federal Institute of Education, Science, and Technology in the Northern region of the country.

D. *Results of the First Set of Tests*

During the labeling phase of the questions, the topics and the main resolution methods were identified. The topics covered in the questions were the theorem of Thales, the Pythagorean theorem, and the similarity of triangles, as well as the use of the rule of three in solving problems.

In some questions, students selected the correct alternative but made errors in the organization of their answers, resulting in point deductions. It is believed that some of these errors are due to lack of attention, especially during the process of transferring calculations from draft sheets to the test.

Among these errors, a notable example is the failure to eliminate the exponent on one side of the equation after moving it to the other side in the form of a square root. Another common mistake was the accidental substitution of a value in the equation, even though the correct value was written immediately afterward, as demonstrated below:

$$X^2 = 120^2 + 90^2$$

$$X^2 = 11400 + 8100$$

$$X^2 = 22500$$

In the previous formula, it can be observed that the highlighted value is incorrect, but the sum is consistent with the value that should be correct, 14400 + 8100. Other errors occurred due to the incorrect interpretation of the questions by the students, as well as a failure to abstract the variables to be used.

For example, in a question about Thales' theorem, the problem could be solved using a proportion, but some students had difficulty constructing the formula and correctly identifying the variables.

Regarding this question, another highlighted error was the partial resolution of the problem. When students found part of the solution, they guessed the second part, with some getting it right but receiving point deductions for not describing how they arrived at the result. This error occurred mainly when calculating the distance between two points, where students found one value of the answer and immediately guessed the second value by approximation. This was observed in 7 out of 11 tests.

Finally, the bonus question, the sixth question, presented more problems in terms of constructing the solution, interpreting the statement, identifying and isolating variables, as well as visualizing and mentally abstracting the problem.

The mentioned question is part of ENEM 2009 but was modified by removing the graphical image that assisted in solving the original problem. The researchers of this study reached a consensus that the absence of this supporting image caused confusion among the students. In most cases, students guessed the answer, with some getting it right. However, they received scores close to zero for not describing how they arrived at the result, or in some cases, the construction of the answer was incorrect, resulting in a significant reduction in scores. Out of the 11 tests, only one student managed to obtain the maximum score on question 6.

After analyzing all the questions of each student, these were mapped and classified, allowing the isolation of the number of errors found by students. A total of 13 errors were mapped, with the two main ones being related to the abstraction of information present in the statements and in the assembly of solutions, caused by a lack of mastery of the formulas that could be used to solve the problems.

Table I presents the mapping data, showing the relationship between the mapped errors and the students who committed them, Table II highlights the description of each error, and Table III shows the relationship of the incorrect questions with the students.

TABLE I. RELATIONSHIP OF ERRORS WITH STUDENTS

Error class	A	B	C	D	K	L	M	N	O	P	R
Error 1	1	0	0	0	0	0	0	0	0	0	0
Error 2	0	0	1	1	1	0	0	0	0	0	0
Error 3	0	2	1	0	1	2	0	1	1	0	0
Error 4	0	1	0	0	0	0	0	0	0	0	0
Error 5	0	0	1	0	0	0	0	0	1	2	3

Error 6	0	0	1	0	0	0	0	0	0	0	0
Error 7	0	0	0	1	0	0	0	1	0	0	0
Error 8	0	0	0	0	1	0	0	0	0	0	0
Error 9	0	0	0	0	0	1	0	0	0	0	0
Error 10	0	0	0	0	0	0	0	1	0	1	0
Error 11	0	0	1	0	0	0	0	0	1	0	1
Error 12	0	0	0	0	0	0	0	0	0	0	1
Error 13	0	0	0	0	0	0	0	0	0	0	1

TABLE II. LIST OF MAPPED ERRORS

Error Class	Description
Error 1	Draft-to-answer transcription error
Error 2	Failure to maintain exponent on one side of the equation after moving it to the other side
Error 3	Formula assembly error
Error 4	Failure to change the sign after moving it to the other side of the equation
Error 5	Failure to abstract the statement
Error 6	Failure to isolate the variables
Error 7	Error in extracting information from figures
Error 8	Lack of organization in calculations
Error 9	Error in finding an answer induced by the distractor
Error 10	Inability to solve the exponent
Error 11	Partial problem resolution and guessing possible answers
Error 12	Division error
Error 13	Replacing multiplication operation with addition

The data presented in Table III have three symbols at the intersections: X for wrong, O for correct, B for not done, and P for partially correct.

TABLE III. RELATIONSHIP OF INCORRECT QUESTIONS WITH STUDENTS

Questions	A	B	C	D	K	L	M	N	O	P	R
Question 1	O	P	P	P	O	P	P	P	P	P	P
Question 2	O	O	O	O	O	X	P	O	X	O	P
Question 3	O	P	X	O	X	O	B	X	O	X	P
Question 4	P	P	O	P	P	O	O	P	X	X	X
Question 5	P	X	X	X	O	O	O	X	O	X	X
Question 6	B	B	X	B	P	P	O	B	B	X	X

E. *Results of the Second Set of Tests*

The analysis of the tests from the second set, related to the exams from the Federal Institute of Education, Science, and Technology of the Northeast of the Country, was conducted in the same way as the previous set, examining each point of the tests to identify and classify errors.

These tests highlighted a significant number of omissions in the non-objective alternatives. In all objective questions, students attempted to respond by selecting one of the alternatives, even if incorrect. However, in the subquestions of question 4, many students left them blank. In total, option A was left blank once, option B was left blank five times, and option C was left blank twice.

A total of 7 errors were mapped in this set of tests. The main type of error was related to problem-solving, with some specific cases showing errors in multiplication, the selection of formulas for problem-solving, and the identification of variable values.

Two other errors were related to a lack of attention in marking the correct answer in the alternatives and partial solution resolution. Table IV respectively relates the test, question, and identified error in its columns.

TABLE IV. MAPPING OF ERRORS IN THE SECOND SET OF TESTS

Test label	Question	Error
2	2	Marked the wrong alternative
2	3	Multiplication error
3	3	Multiplication error
9	4 – C	Partial resolution of the problem
10	4 – A	Error in assembling the formula
10	4 – B	Error in problem-solving
10	4 – C	Failure to isolate variables

F. *Results of the Third Set of Tests*

The third set of tests was also analyzed along with the previous ones, and the results indicated errors in operations, errors in swapping or incorrectly using signs, mainly related to transferring the value from one side to the other of the equation. Partial response errors were also very common, especially linked to the fact that students found only 1 of the 2 or 2 of the 4 roots of the questions.

Errors in interpreting the questions are related to a lack of understanding or an inability to isolate terms and identify the means of resolution. Thus, these students used terms incorrectly or chose formulas that did not satisfy the problem. Another error, in a similar order of occurrence to the previous one, was the operation error. This error occurred when students were unable to perform divisions and subtractions in the questions, resulting in incorrect answers. Table V presents these errors and their relationships with the questions and tests of the third set of tests.

IV. DISCUSSION

The generated data indicate common errors among students, such as difficulties in constructing solutions and interpreting problem statements, as well as occasional errors in transcribing answers. In general, two main groups of errors can be identified: one associated with distractors and another with a lack of content mastery. It is important to note that belonging to one group does not exclude the possibility of belonging to the other, as some students fall into both categories.

TABLE V. MAPPING OF ERRORS IN THE THIRD SET OF TESTS

Error	Questions (Test)
Operation error (subtraction and/or division)	2(1), 10(7) and 3(11)
Partial answer	3(1), 1(5), 2(5), 4(5), 1(6), 3(6), 4(6), 1(8), 2(8), 3(8), 3(9), 2(13), 3(13) and 3(16)

Sign swap	3(2), 3(5), 1(6), 8(6), 1(7), 10(7), 1(8), 6(8), 10(9), 1(10), 3(10), 1(11), 2(14), 9(14), 3(15) and 4(15)
Random choice of answer	5(7) and 6(11)
Error in using the Bhaskara formula	9(7)
Interpretation error	4(11), 1(12) and 1(13)
Left blank	3(12), 4(12), 1(14), 1(16) and 4(16)

Among the errors highlighted in Tables II, IV and V, errors in division, subtraction, and multiplication were identified. These errors can be classified as a single type, linked to a lack of mastery of basic calculations, classified as an operational error.

It is also noticeable that distraction errors were committed in the sets of tests, in addition to partial resolution. The first error is characterized as a distraction error, and the second error is linked to complete content mastery since, after analyzing the formulas and paths used, it does not seem that these questions were left incomplete only due to the short time for solving the problems. This characteristic arises because part of the solutions was correct; however, they were abandoned in parts that students could not solve.

Another common error among the sets of tests was the identification of variables in the statements. This error seems to be linked to a lack of complete mastery of the abstraction of elements from reading, an interdisciplinary problem that needs attention, as it can generate a lot of difficulty for these students not only in learning but in other aspects of their lives. This issue deserves attention whenever identified, allowing educators to validate whether the problem is real and also to investigate its causes more deeply.

An important point to analyze is that those who do not have complete mastery of the subjects may have difficulties associated with a phenomenon linked to COVID-19. Numerous students were affected during the pandemic, and even those who were promoted may have difficulties in learning, consequently creating challenges in subsequent subjects and grades in education [7].

Note that this study raised specific cases of errors, seeking to map the smallest parts of errors, such as subtraction, addition, errors in value passage in equations, among others, also observing subjective factors such as distractions and misconceptions. From this analysis, it is possible to identify general classes of errors that can be associated with specific situations where their occurrence is noted.

In this sense, if a student makes an error in a first-degree equation problem, it is possible to map the error within the mapped cases, allowing digital systems to make this association, including being another type of data that can be used in intelligent systems.

In addition to the above, these fragmented errors to their reducible versions, that is, the smallest present error cause, can serve to recreate more complex errors. An addition error in a simple addition task may indicate more than the addition error itself; it may indicate subjective factors, such as those already mentioned, or other affective states and causes; however, the perceivable error was indeed the operation error, which may hide other problems that interfere or interfered with the student's learning.

In this way, computational systems can, in addition to identifying causes, also suggest more complex analyses of the student's overall learning state. Another context is in solving complex problems. Generally, when solving complex mathematical problems, it is expected that one has mastery of basic mathematical operations, but carelessness or indeed an error can occur, allowing the system to present the specific point of error, reducing frustrations and allowing the construction of knowledge about the reflection of where the error occurred.

Finally, another point of reflection on the use of this methodology for building databases on mathematical errors is the possibility of scaling problems, classifying them, and also specifying situations. Thus, from the combination of two or more errors, it is possible to build a tree of situations that may have led to the identified error(s). In this context, the database functions as a general base that can be associated with more complex identification situations. Within the analyzed context itself, we can highlight distraction errors, which can be associated with sign changes, for example, but can also be linked to incomplete mastery of this content. Once mapped, given the context in which the test was applied, the student's profile, and other complementary information, it is possible to apply intelligent systems to seek to provide better feedback, both for the student and for teachers.

In addition to the points mentioned as benefits and contributions of this study, the creation of the database itself also stands out, something that is usually difficult to access, with each research team creating its own with its own standard. From a general database of errors, and the general association of error causes, in addition to the specific identification of this error step by step, for example, it becomes very beneficial to have a shared database of common errors and atoms for the creation of complex support systems in the teaching and learning of mathematics, providing not the final solution, but the basis for each team to start from general data and thus be able to build their specific and more complex solutions.

In this context, a suggested approach is the creation of Intelligent Tutoring Systems capable of identifying, even if only having access to the problem and the student's response, step-by-step solutions to this provided problem, building the identification of the error and possible causes and providing personalized feedback for students and teachers. This approach can benefit students who are somewhat reluctant to ask questions after making a mistake or not fully understanding where they went wrong, as well as provide the teacher with a complementary and decision-making view from the analysis of the ITS, potentially favoring their mechanisms of action with this student. The Table VI presents the suggested error base.

TABLE VI. ERROR BASE SUGGESTED

Class of errors	Errors
Operation Errors	Replacement of multiplication operation with addition

	Division error
	Multiplication error
	Subtraction and/or division operation error
	Sign swap
Formula and Equation Errors	Error in assembling the formula
	Failure to maintain exponent on one side of the equation after moving it to the other side
	Failure to change the sign after moving it to the other side of the equation
	Failure to isolate the variables
	Failure to abstract the statement
	Failure to isolate the variables
Information Processing Errors	Error in extracting information from figures
	Lack of organization in calculations
	Interpretation error
Problem-solving Errors	Partial problem resolution and guessing possible answers
	Error in problem-solving
	Partial answer
Alternative Selection Errors	Marked the wrong alternative
	Random choice of answer
	Left blank
Domain and Formula Usage Errors	Inability to solve the exponent
	Error in using the Bhaskara formula

This discussion sought to present an overview of the results obtained and the benefits and contributions that this study raised, associating these findings with their application to ITS.

V. Threats to Validity

Several threats to the validity of this study have been identified, and they will be cited and contextualized below.

The study group was considered ideal for this error mapping since they were students from basic education, from the so-called Middle Level in Brazil, meaning they are students who are learning more complex concepts and should master arithmetic, providing a good dataset from the analysis of their tests. However, some data on age and gender could not be collected due to decisions by some teachers who provided the tests. Thus, despite presenting significant contributions, the study could have provided specific data on the demographic and gender of the students, providing an even more detailed and accurate insight for the construction of ITS focused on mathematics and the overall understanding of the education of these students.

Another threat to validity is associated with the quantity of provided tests, as only tests containing errors were provided in one of the schools, and the exact number of students in the class was not provided. It was necessary to infer this number by relating it to the average number of students attending basic education in public schools in Brazil, which is usually around 50 students, although there are political proposals to limit this number to 25 or 35 students. Although this limitation does not harm the presented data, it makes the identification of the percentage of errors occurring in the class less precise.

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VI. Conclusion

This exploratory study aimed to map common errors in mathematics, allowing computer systems to use this data for error analysis and personalized feedback. In this sense, this study investigated tests from 63 students from two public educational institutions, one federal and one state institution in Brazil. The results indicated that it is possible to map, identify, and classify common student errors from tests and thus identify indivisible sets of errors, such as multiplication errors and sign changes, as well as macro errors, such as incorrect use of formulas, and adverse causes, such as distractions and other issues.

As expected, some threats to validity are also present in this study, such as the fact that socioeconomic factors were not recorded, as well as the small number of tests. Despite that, the data support the research hypotheses of this work, indicating that mapping mathematical problems, despite being a complex and costly task, is an important path for creating robust and reliable bases for the creation of tutoring systems applied to this discipline.

As future work, it is expected to apply these findings in an ITS and validate it in school classes, as well as to extend this study by analyzing more tests in different educational levels.

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